High School Integrated Math II Blueprint

The focus of Mathematics II is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics I. The need for extending the set of rational numbers arises and real and complex numbers are introduced to that all quadratic equations ban be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with the quadratic algebraic representations round out the course. The Mathematical Practice standards apply throughout Math II and together with the content standards prescribe that students experience math as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Suggested Quarter/Time	Instructional Focus 1 (IM2IF1)	CCSS Mathematical Content	CCSS Mathematical Practice	Content
1st/10 days	**Extending the Number System**	**Extend the properties of exponents to rational exponents.** N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.) N.RN.2 - Rewrite expressions involving radicals and rational exponents using the properties of exponents. **Use properties of rational and irrational numbers** N.RN.3 - Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. *Connect to physical situations, e.g., finding the perimeter of a square of area 2.* **Perform arithmetic operations with complex numbers.** N.CN.1 **-** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. N.CN.2 **-** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **Perform arithmetic operations on polynomials.**	**Directly addressed practices are underlined** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics**.** 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	**myOER:** Modeling Rolling Cups Imaginary Numbers? What do you Mean Imaginary? Lesson examples above address at least one (or more) of the Common Core State Standards included in this Instructional Focus. To find more lessons for this instructional focus, please use the **Advanced Search** and type the keyword – **IM2IF1**. **Teacher:**

		A.APR.1 - Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. *Focus on polynomial operations that simplify to forms that are linear or quadratic in a positive integer power of x.*		

Suggested Quarter/Time	Instructional Focus 2 (IM2IF2)	CCSS Mathematical Content	CCSS Mathematical Practice	Content
1st-2nd/36 days	**Quadratic Functions and Modeling**	**Interpret functions that arise in applications in terms of a context** **F.IF.4** - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. *Focus on quadratic functions; compare with linear and exponential functions studied in unit 2.* **F.IF.5** - Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **F.IF.6** - Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **Analyze functions using different representations** **F.IF.7** - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *Compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions.*	**Directly addressed practices are underlined** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	**myOER:** Chutes and ladders - Quadratic equations review Building Connections Lesson examples above address at least one (or more) of the Common Core State Standards included in this Instructional Focus. To find more lessons for this instructional focus, please use the **Advanced Search** and type the keyword – **IM2IF2.** **Teacher:**

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.IF.8 - Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
c. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

F.IF.9 - Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. *Focus on expanding the types of functions considered to include, linear, exponential, and quadratic. Include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.*

Build a function that models a relationship between two quantities

F.BF.1 - Write a function that describes a relationship between two quantities. *Focus on situations that exhibit a quadratic relationship.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Build new functions from existing functions

F.BF.3 - Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the

		value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *Focus on quadratic functions, and consider including absolute value functions.* **F.BF.4** - Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or f(x) = (x+1)/(x-1) for x ≠ 1. *Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x)=x^2$, x>0.* **Construct and compare linear, quadratic, and exponential models and solve problems** **F.LE.3** - Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. *Compare linear and exponential growth to quadratic growth.*		

Suggested Quarter/Time	Instructional Focus 3 (IM2IF3)	CCSS Mathematical Content	CCSS Mathematical Practice	Content
2nd/24 days	**Expressions and Equations**	**Interpret the structure of expressions** **A.SSE.1** - Interpret expressions that represent a quantity in terms of its context. *Focus on quadratic and exponential expressions.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. *Exponents are extended from the integer exponents found in Unit 1 to rational exponents focusing on those that represent square or cube roots.* **A.SSE.2** - Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. **Write expressions in equivalent forms to solve**	**Directly addressed practices are underlined** 1. Make sense of problems and persevere in solving them. 2. <u>Reason abstractly and quantitatively.</u> 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. <u>Attend to precision.</u> 7. <u>Look for and make use of structure.</u> 8. Look for and express regularity in repeated	**myOER:** Forming Quadratics Solving Quadratic Equations by Factoring Lesson examples above address at least one (or more) of the Common Core State Standards included in this Instructional Focus. To find more lessons for this instructional focus, please use the **Advanced Search** and type the keyword – **IM2IF3.** **Teacher:**

		problems **A.SSE.3** - Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.* *a.* Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. **Create equations that describe numbers or relationships.** **A.CED.1 -** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **A.CED.2** - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A.CED.4** - Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R. Extend to formulas involving squared variables.* **Solve equations and inequalities in one variable.** **A.REI.4** - Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for	reasoning.	

		$x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. *Extend to solving any quadratic equation with real coefficients including those with complex solutions.* **Use complex numbers in polynomial identities and equations.** **N.CN.7** - Solve quadratic equations with real coefficients that have complex solutions. **N.CN.8 (+)** Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. **N.CN.9 (+)** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **Solve systems of equations.** **A.REI.7 -** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.* *Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions.*		

Suggested Quarter/Time	Instructional Focus 4 (IM2IF4)	CCSS Mathematical Content	CCSS Mathematical Practice	Content
2nd-3rd/20 days	**Applications of Probability**	**Understand independence and conditional probability and use them to interpret data.** **S.CP.1** - Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **S.CP.2** - Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **S.CP.3** - Understand the conditional probability of A given B as P(A and B)/P(B), and interpret	**Directly addressed practices are underlined** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically.	**myOER:** Modeling Conditional Probabilities 2 Pennies, Pennies, and More Pennies Lesson examples above address at least one (or more) of the Common Core State Standards included in this Instructional Focus. To find more lessons for this instructional

		independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. **S.CP.4** - Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* **S.CP.5** - Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* **Use the rules or probability to compute probabilities of compound events in a uniform probability model.** **S.CP.6** - Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. **S.CP.7** - Apply the Addition Rule, P(A or B) = (A) + P(B) – P(A and B), and interpret the answer in terms of the model. **S.CP.8 (+)** Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B/A) = P(B)P(A|B), and interpret the answer in terms of the model. **S.CP.9 (+)** Use permutations and combinations to compute probabilities of compound events and solve problems. **Use probability to evaluate outcomes of decisions.** **S.MD.6 (+)** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **S.MD.7 (+)** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	focus, please use the **Advanced Search** and type the keyword **– IM2IF4.** **Teacher:**

Suggested Quarter/Time	Instructional Focus 5 (IM2IF5)	CCSS Mathematical Content	CCSS Mathematical Practice	Content
3rd-4th/45 days	**Similarity, Right Triangle Trigonometry, and Proof**	**Understand similarity in terms of similarity transformations.** **G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G.SRT.2** - Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G.SRT.3** - Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **Prove geometric theorems.** **G.CO.9** - Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. **G.CO.10** - Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. **G.CO.11** - Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. *Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without*	**Directly addressed practices are underlined** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	**myOER:** Postulates and proofs: Let's take it to the courtroom! Geometry Problems: Circles and Triangles Lesson examples above address at least one (or more) of the Common Core State Standards included in this Instructional Focus. To find more lessons for this instructional focus, please use the **Advanced Search** and type the keyword **– IM2IF5.** **Teacher:**

		words. Focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. **Prove theorems involving similarity.** **G.SRT.4** - Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. **G.SRT.5** - Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **Use coordinates to prove simple geometric theorems algebraically.** **G.GPE.6** - Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **Define trigonometric ratios and solve problems involving right triangles.** **G.SRT.6** - Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G.SRT.7** - Explain and use the relationship between the sine and cosine of complementary angles. **G.SRT.8** - Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **Prove and apply trigonometric identities.** **F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ), given sin(θ), cos(θ), or tan(θ), and the quadrant of the angle. *Limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula.*		

Suggested Quarter/Time	Instructional Focus 6 (IM2IF6)	CCSS Mathematical Content	CCSS Mathematical Practice	Content
4th/20 days	**Circles With and Without Coordinates**	**Understand and apply theorems about circles.** **G.C.1** - Prove that all circles are similar. **G.C.2** - Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and	**Directly addressed practices are underlined** 1. Make sense of problems and persevere in solving	**myOER:** Solving Problems Involving Chords, Radii, Tangents,

		circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. **G.C.3** - Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **G.C.4 (+)** Construct a tangent line from a point outside a given circle to the circle. **Find arc lengths and areas of sectors of circles.** **G.C.5** - Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. *Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure.* **Translate between the geometric description and the equation for a conic section.** **G.GPE.1** - Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **G.GPE.2** - Derive the equation of a parabola given a focus and directrix. *Connect the equations of circles and parabolas to prior work with quadratic equations. The directrix should be parallel to a coordinate axis.* **Use coordinates to prove simple geometric theorems algebraically.** **G.GPE.4** - Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).* **Explain volume formulas and use them to solve problems.** **G.GMD.1** - Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. **G.GMD.3** - Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Secants and Arcs within the Same Circle Calculating Volumes of Compound Objects Lesson examples above address at least one (or more) of the Common Core State Standards included in this Instructional Focus. To find more lessons for this instructional focus, please use the **Advanced Search** and type the keyword **– IM2IF6.** **Teacher:**

		Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.		

Content-myOER:

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Adapted from The Charles A. Dana Center at the University of Texas at Austin; CommonCoreTools.me by Bill McCallum; and Common Core State Standards for Mathematics, http://doe.sd.gov/board/packets/documents/Sept10/CommonCore_document3.pdf, http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf